

In the succeeding pages some of these theories will be outlined. First the principles of probability theory, basic for any analysis of operational systems, and next a straightforward application of probability, search theory, is discussed. Next come two chapters on the dynamics of a large class of simple operations, which are characterized by being, at any instant, in one or another of a discrete set of states, with transitions from state to state expressible in terms of probability. If the transitions can be considered to occur only at the ends of discrete time intervals, the equations are those of the Markov process; if the transitions may take place at any time, the equations are those of Kolmogorov. For this class the analogy is fairly close to the equations of classical dynamics, with the random influences from outside being the analogues of the external forces and the specified rules of operation the analogues of the internal forces of the mechanical system, both being represented by various elements of the transition probability matrix. In some cases the random fluctuations of the internal forces can be neglected, and the operational system can be represented in terms of a control-system model, as is shown in Chapter 6.

In more complex systems the complete solution for the behavior of the system cannot be obtained in analytical terms, and one must determine optimal rules of operation by sequential methods, as is indicated in Chapter 8. Special kinds of operations serve as examples of the use of the foregoing techniques, in Chapters 9 to 11. In still more complicated cases, the use of the digital computer to simulate the operation may be the most effective way to gain an understanding of the operation, as is shown in the last chapter.

NOTES ON OPERATIONS RESEARCH

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PREFACE

For the past six years the Operations Research Center at the Massachusetts Institute of Technology has given special summer programs of lectures on topics in operations research for persons from industry, from the military services, and from governmental departments. For use in these programs a selection of material on recent developments in the field has been collected from time to time and published in offset form by the Technology Press.

The occasion for the publication of this latest selection is the scheduling of a Special Program in Operations Research, for persons from European NATO countries, at the Center for Experimental Aerodynamics in Brussels in August, 1959. For the past few years an increasing number of persons from Europe have attended our regular summer programs in Cambridge, Massachusetts. At the request of several NATO organizations, it was decided to schedule an additional Program in Europe in 1959, for the convenience of the Europeans interested in the subject. Support for the additional expenses entailed has been provided by the U. S. Air Force, under contract AF19(604)-5442. It is hoped that this program will stimulate the future scheduling of similar programs, in the various NATO countries, to be conducted by European experts in this rapidly growing field. The present Notes are being published in somewhat more convenient form, expecting that they may be of use in these other programs, as well as in the ones still to be scheduled at the Massachusetts Institute of Technology.

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These Notes make no pretense of covering all the aspects of Operations Research, nor even of complete consistency among the contributions here collected. At the present stage of rapid development of the field it was more appropriate to make available quickly some of the recent advances, rather than to spend several years in producing a complete text with more unified point of view. Publication deadlines have prevented the exhaustive cross-checking of formulas needed to remove all errors and misprints. We hope that the errors which occur will not be serious ones.

The nature of the Program, which these notes supplement, accounts for the predominantly theoretical nature of the subjects covered. A two-week series of lectures may hope to give the prepared participant a preliminary insight into some of the newer techniques of the mathematical representation of operations; it cannot hope to train the participant in the equally important techniques of data gathering and operational experiment. This program, and these Notes, are not intended to create operations researchers ab initio; they are intended to provide one who already has some O. R. experience with additional theoretical tools to use in his future work. Within these limitations, it is hoped these Notes will be of use.

April, 1959

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INTRODUCTION

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An operation is a pattern of activity of men or of men and machines, engaged in carrying out a cooperative and usually repetitive task, with pre-set goals and according to specified rules of operation. The scientific study of operations is called operations research. Because men are involved in operations, a considerable amount of variability occurs between samples of similar operations; but because of their repetitive nature these variations can be systematized by the use of probability theory. By studying similarities of pattern, predictions about one operation can often be made from studies of other operations, isomorphic with the first; and thus the concept of "repetitive nature" can be generalized.

Operations research is not just an exercise in probability theory, however. It is a scientific study in its own right, using experimental as well as theoretical techniques to study a natural phenomenon, an operation. Its object is to understand the behavior of various operations, by the appropriate use of observation, controlled experimentation and theoretical analysis, so as to predict the operational result of changes in operating rules or in equipment, and thus better to control the operation and improve its result.

This increased understanding and resulting improvement in control are of immediate practical value to the managers of industrial, military, or governmental operations; hence, much work in applied operations research is done at the request of, and in close collaboration with, management. Such close collaboration is usually necessary in order to carry out valid operational experiments and in order to gain a clear insight into the reasons for existing rules of operation.

A course in operations research, carried out away from an actual operation, is liable to engender the same misconceptions which arise when any experimental science is taught away from the laboratory. Lectures, such as those outlined in the following pages, inevitably emphasize theories and mathematical techniques; the other half of the work of the operations researcher, the experimental aspects, cannot so easily be taught by lectures and textbooks, it must be learned by doing. And so the reader should continually bear in mind that the theories discussed in this book are of no avail at all unless they are used on valid data, based on careful observation, and that one who has mastered the mathematics here contained is not yet a qualified operations researcher until he has learned how to plan and carry out operational experiments.
